Oxford Cambridge and RSA

Accredited

A Level Further Mathematics B (MEI)

Y431 Mechanics Minor

Sample Question Paper

Date – Morning/Afternoon

Time allowed: 1 hour 15 minutes

OCR supplied materials:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)
- Scientific or graphical calculator

MODEL SOLUTIONS

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.**
- Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

1 In this question, **i** and **j** are perpendicular unit vectors in a horizontal plane.

A particle P has mass 10 kg and a speed of $20\,\mathrm{m\,s^{-1}}$ in the direction of $4\mathbf{i} + 3\mathbf{j}$. A force of $(-4\mathbf{i} + 15\mathbf{j})$N acts on P for 8 seconds.

(i) Calculate the impulse of the force over the 8 seconds. **[1]**

$I = F\Delta t$

$= (-4i + 15j)\,8$

$= \underline{-32i + 120j}$ Ns

(ii) Hence find the speed of P at the end of the 8 seconds. **[3]**

Initial $v = \dfrac{20}{\sqrt{4^2+3^2}}(4i+3j)$

$= 4(4i+3j) = \underline{16i + 12j}$ ms^{-1}

$I = m(v-u) \Rightarrow -32i + 120j = 10(v - 16i - 12j)$

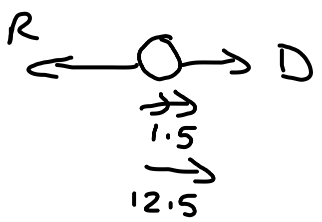
$\Rightarrow -32i + 120j = 10v - 160i - 120j$

$\Rightarrow v = 12.8i + 24j$

Speed $= |v| = \sqrt{12.8^2 + 24^2} = \underline{27.2\,\mathrm{ms^{-1}}}$

2 A car of mass 1200 kg is travelling in a straight line along a horizontal road. At a time when the power of the driving force is 25 kW, the car has a speed of $12.5\,\mathrm{m\,s^{-1}}$ and is accelerating at $1.5\,\mathrm{m\,s^{-2}}$.

Calculate the magnitude of the resistance to the motion of the car. **[5]**



$P = Fv = Dv$

$\Rightarrow 25000 = D \times 12.5$

$\Rightarrow D = 2000\,N$

N2L on Car: $D - R = 1.5(1200)$

$\Rightarrow 2000 - R = 1800$

$\Rightarrow R = \underline{200\,N}$

3 **(i)** Find the dimensions of
- density and
- pressure (force per unit area). **[2]**

$[\text{density}] = \underline{\underline{ML^{-3}}}$

$[\text{pressure}] = \left[\frac{F}{A}\right] = \frac{MLT^{-2}}{L^2} = \underline{\underline{ML^{-1}T^{-2}}}$

The frequency, f, of the note emitted by an air horn is modelled as $f = ks^{\alpha}p^{\beta}d^{\gamma}$, where
- s is the length of the horn,
- p is the air pressure,
- d is the air density,
- k is a dimensionless constant.

(ii) Determine the values of α, β and γ. **[4]**

$[f] = [s]^{\alpha}[p]^{\beta}[d]^{\gamma}$

$\Rightarrow T^{-1} = L^{\alpha}(ML^{-1}T^{-2})^{\beta}(ML^{-3})^{\gamma}$

$T: \quad -1 = -2\beta \Rightarrow \beta = \frac{1}{2}$

$L: \quad 0 = \alpha - \beta - 3\gamma \Rightarrow \frac{1}{2} = \alpha - 3\gamma$

$M: \quad 0 = \beta + \gamma \Rightarrow \gamma = -\frac{1}{2} \Rightarrow$ Subs in L:

$\Rightarrow \frac{1}{2} = \alpha + \frac{3}{2} \Rightarrow \alpha = -1$ $\quad \boxed{\therefore \alpha = -1,\ \beta = \frac{1}{2},\ \gamma = -\frac{1}{2}}$

A particular air horn emits a note at a frequency of 512Hz and the air pressure and air density are recorded. At another time it is found that the air pressure has fallen by 2% and the air density has risen by 1%. The length of the horn is unchanged.

(iii) Calculate the new frequency predicted by the model. **[2]**

$512 = ks_0^{-1}\sqrt{\frac{p_0}{d_0}}$

$f_1 = ks_0^{-1}\sqrt{\frac{0.98p_0}{1.01d_0}} = \sqrt{\frac{0.98}{1.01}} \times \left(ks_0^{-1}\sqrt{\frac{p_0}{d_0}}\right)$

$= \sqrt{\frac{0.98}{1.01}}(512) = \underline{\underline{504.3\text{Hz}}}$

4 Fig. 4 shows a non-uniform rigid plank AB of weight 900 N and length 2.5 m. The centre of mass of the plank is at G which is 2 m from A. The end A rests on rough horizontal ground and does not slip. The plank is held in equilibrium at 20° above the horizontal by a force of T N applied at B at an angle of 55° above the horizontal as shown in Fig. 4.

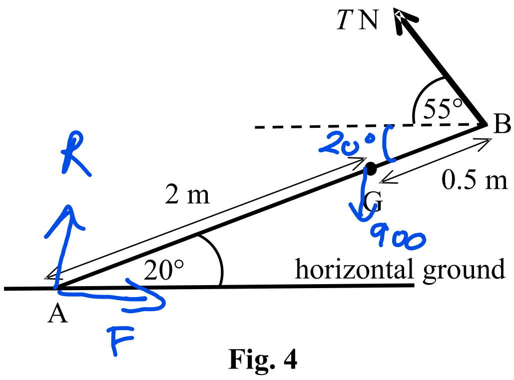


Fig. 4

(i) Show that $T = 700$ (correct to 3 significant figures). **[4]**

A: $900\cos 20 \times 2 = T\sin(20+55) \times 2.5$

$\Rightarrow T = \dfrac{1800\cos 20}{2.5\sin(75)}$

$\Rightarrow T = 700.445\ldots N \approx \underline{\underline{700N}}$ (3 s.f.)

(ii) Determine the possible values of the coefficient of friction between the plank and the ground. **[5]**

Resolving $\updownarrow$: $R + T\sin 55 = 900$

$R = 900 - 700\sin 55 = \underline{\underline{326.23N}}$

Resolving $\leftrightarrow$: $F = T\cos 55 = 700\cos 55 = \underline{\underline{401.76N}}$

$F \le \mu R \Rightarrow 401.759 \le \mu \times (326.23)$

$\Rightarrow \underline{\underline{\mu \ge 1.23}}$

5 A young man of mass 60 kg swings on a trapeze. A simple model of this situation is as follows.

The trapeze is a light seat suspended from a fixed point by a light inextensible rope. The man's centre of mass, G, moves on an arc of a circle of radius 9 m with centre O, as shown in Fig. 5. The point C is 9 m vertically below O. B is a point on the arc where angle COB is 45°.

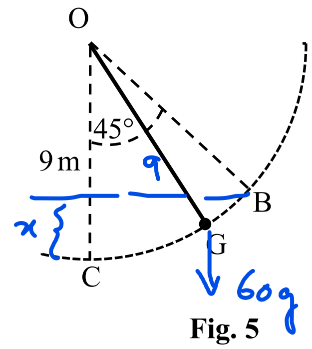
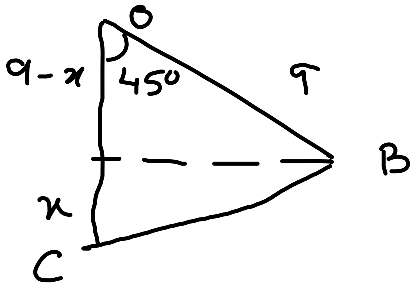


Fig. 5

(i) Calculate the gravitational potential energy lost by the man if he swings from B to C. **[3]**



$$\cos 45 = \frac{9-x}{9}$$

$$x = 9 - 9\cos 45$$

$$x = 9(1-\cos 45)$$

GPE lost $= mg\Delta h = mgx$

$$= 60 \times 9.8 \times 9\left(1 - \frac{\sqrt{2}}{2}\right) = 1549.99\ldots \text{ J}$$

$$= \underline{1550 \text{ J}} \quad (3\text{sf.})$$

In this model it is also assumed that there is no resistance to the man's motion and he starts at rest from B.

(ii) Using an energy method, find the man's speed at C. **[2]**

Conservation of Energy:

$$\Delta KE = \Delta GPE$$

$$\frac{1}{2}mv^2 = 1550$$

$$\frac{1}{2} \times 60 \times v^2 = 1550$$

$$v^2 = 51.\dot{6}\ldots$$

$$v = \underline{7.19 \text{ ms}^{-1}}$$

A new model is proposed which also takes into account resistance to the man's motion.

(iii) State whether you would expect any such model to give a larger, smaller or the same value for the man's speed at C. Give a reason for your answer. **[1]**

Smaller, because he will have done work against resistive forces and so his KE at the bottom will be lower.

A particular model takes account of the resistance by assuming that there is a force of constant magnitude 15 N always acting in the direction opposing the man's motion. This new model also takes account of the man 'pushing off' along the arc from B to C with a speed of 1.5 m s^{-1}.

(iv) Using an energy method, find the man's speed at C. **[5]**

KE at B $= \frac{1}{2} \times 60 \times 1.5^2 = \underline{67.5\text{ J}}$

WD against res. $= 15 \times$ arc length $= 15 \times \theta r$

$= 15 \times \frac{\pi}{4} \times 9 = \underline{\underline{\frac{135}{4}\pi \text{ J}}}$

$\frac{1}{2} \times 60 \times v^2 - 67.5 = 1550 - \frac{135}{4}\pi$

$\Rightarrow 30v^2 = 1511.46\ldots$

$\Rightarrow v = 7.098 \text{ ms}^{-1} \approx \underline{\underline{7.10 \text{ ms}^{-1}}}$ (3 s.f.)

6 My cat Jeoffry has a mass of 4kg and is sitting on rough ground near a sledge of mass 8kg. The sledge is on a large area of smooth horizontal ice.

Initially, the sledge is at rest and Jeoffry jumps and lands on it with a horizontal velocity of 2.25m s^{-1} parallel to the runners of the sledge. On landing, Jeoffry grips the sledge with his claws so that he does not move relative to the sledge in the subsequent motion.

(i) Show that the sledge with Jeoffry on it moves off with a speed of 0.75m s^{-1}. **[2]**

$\xrightarrow{+}$ Before: (G) 4g, $2.25 \rightarrow$ (S) 8g, $0 \rightarrow$

After: ○ 12g, $v \rightarrow$

PCLM: $2.25 \times 4 + 0 = v \times 12$

$v = \underline{\underline{0.75 \text{ ms}^{-1}}}$

With the sledge and Jeoffry moving at $0.75\,m\,s^{-1}$, the sledge collides *directly* with a stationary stone of mass 3 kg. The stone may move freely over the ice. The coefficient of restitution in the collision is $\frac{4}{15}$.

(ii) Calculate the velocity of the sledge and Jeoffry immediately after the collision. **[6]**

$\xrightarrow{+}$ Before: G $\rightarrow 0.75$, 12kg   S $0 \rightarrow$, 3kg

After: G $\rightarrow V_G$, 12kg   S $V_S \rightarrow$, 3kg

$$e = \frac{V_S - V_G}{0.75 - 0} \Rightarrow \frac{4}{15} = \frac{V_S - V_G}{0.75}$$

$$\Rightarrow 0.2 = V_S - V_G \Rightarrow V_S = 0.2 + V_G \quad (1)$$

PCLM: $0.75(12) + 0 = 12V_G + 3V_S$

$$\Rightarrow 9 = 12V_G + 3V_S \quad (2)$$

Subs (1) in (2): $9 = 12V_G + 3(0.2 + V_G)$

$$\Rightarrow 8.4 = 15V_G$$

$$\Rightarrow V_G = \underline{\underline{0.56\ ms^{-1}}}$$

In a new situation, Jeoffry is initially sitting at rest on the sledge when it is stationary on the ice. He then walks from the back to the front of the sledge.

(iii) Giving a brief reason for your answer, describe what happens to the sledge during his walk. **[2]**

The sledge will move in opposite direction to Jeoffry. This is because there are no external forces acting on them, so linear momentum will be conserved.

Jeoffry is again sitting on the sledge when it is stationary on the ice. He jumps off and, after he has lost contact with the sledge, has a horizontal speed relative to the sledge of $3\,m\,s^{-1}$.

(iv) Determine the speed of the sledge after Jeoffry loses contact with it. **[4]**

$\xrightarrow{+}$ Before: O $\rightarrow 0$, 12kg

After: $v \leftarrow$ 8 kg $\quad$ 4 $\rightarrow u$

$u + v = 3 \Rightarrow u = 3 - v$ — (i)

PCLM: $0 = 4u - 8v$

$\Rightarrow 8v = 4u$

$\Rightarrow 2v = u$

Subs (i): $2v = 3 - v$

$\Rightarrow v = \underline{\underline{1\,\text{ms}^{-1}}}$

7 Fig. 7 shows a container for flowers which is a vertical cylindrical shell with a closed horizontal base. Its radius and its height are both $\frac{1}{2}$ m. Both the curved surface and the base are made of the same thin uniform material. The mass of the container is M kg.

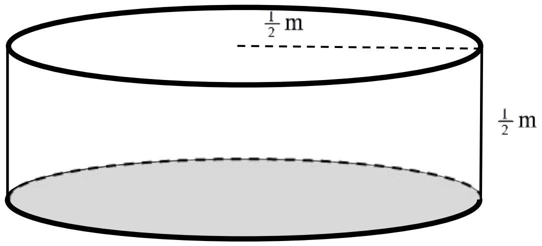


Fig. 7

(i) Find, as a fraction, the height above the base of the centre of mass of the container. **[3]**

$\bar{y}$ is dist. of C.o.M above the base:

Area of base $= \pi(0.5)^2 = \pi/4$

" " Side $= \pi/2$

Dist. of C.o.M of side from base $= \frac{\frac{1}{2}}{2} = 1/4$

$$\bar{y}\left(\frac{\pi}{4} + \frac{\pi}{2}\right) = 0\left(\frac{\pi}{4}\right) + \frac{1}{4}\left(\frac{\pi}{2}\right)$$

$$\frac{3\cancel{\pi}}{4}\bar{y} = \frac{\cancel{\pi}}{8}$$

$$\bar{y} = \frac{4}{24} = \underline{\underline{1/6}}$$

The container would hold $\frac{3}{2}M$ kg of soil when full to the top.

Some soil is put into the empty container and levelled with its top surface y m above the base. The centre of mass of the container with this much soil is zm above the base.

(ii) Show that $z = \frac{1+9y^2}{6(1+3y)}$. **[4]**

C.O.M of soil $= y/2$

Mass of soil $= \frac{3M}{2} \times \frac{y}{1/2} = 3My$

" " base $= \frac{M \times \pi/4}{\pi/4 + \pi/2} = M/3$

" " Side $= \frac{M \times \pi/2}{\pi/4 + \pi/2} = 2M/3$

$z\left(3My + \frac{M}{3} + \frac{2}{3}M\right) = 3My\left(\frac{y}{2}\right) + \frac{M}{3}(0) + \frac{2}{3}M\left(\frac{1}{4}\right)$

$z(3My + M) = \frac{3My^2}{2} + \frac{M}{6}$

$z(3y+1) = \frac{3y^2}{2} + \frac{1}{6} \Rightarrow z(3y+1) = \frac{9y^2+1}{6}$

$\Rightarrow z = \frac{9y^2+1}{6(3y+1)}$ // shown.

(iii) It is given that $\frac{dz}{dy} = 0$ when $y = 0.14$ (to 2 significant figures) and that $\frac{d^2z}{dy^2} > 0$ at this value of y.

When putting in the soil, how might you use this information if the container is to be placed on slopes without it tipping over? **[2]**

This says that when $y = 0.14$, the C.o.M is the lowest.

So, at this value, the container can be placed at the steepest value without it tilting.

END OF QUESTION PAPER